Technical Note: a simplified model for the differences between daytime and nighttime LYRA occultation measurements

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As was shown in the presentation of Joe Zender's presentation, there are systematic differences between sunset and sunrise LYRA measurements. A simple calculation explains the characteristics of these differences.

1 Assumptions

We assume:

- The Sun is a point source.
- Homogeneous atmospheric shells: number density of species only depends on altitude: $n(z)$.
- A systematic difference between the number density of a certain (unspecified) constituent at local sunset and sunrise: $n_{\mathrm{sunset}}(z)$ and $n_{\mathrm{sunrise}}(z)$.
- Absorption or scattering cross section σ that is independent of temperature and pressure (and therefore independent of altitude).
- Monochromatic Lyra channels

2 The signal

In each LYRA channel at wavelength λ, the signal is then given by the law of Beert-Lambert (for the optical path with tangent point altitude z_{t}):

$$I(z_{\mathrm{t}}, \lambda) = I_0(\lambda) \exp\left[-\int_{\mathrm{Sun}}^{\mathrm{Sat}} \sigma(\lambda) n(z(s))\,\mathrm{d}s\right]$$

with s the parameter describing length along the optical path. If we use the *tangent line integrated density*, associated with the optical path:

$$N(z_{\mathrm{t}}) = \int_{\mathrm{Sun}}^{\mathrm{Sat}} n(z(s))\,\mathrm{d}s$$

then the modelized signal is:

$$I(z_{\mathrm{t}}, \lambda) = I_0(\lambda) \exp\left[-\sigma(\lambda) N(z_{\mathrm{t}})\right]$$

Using the optical thickness along the optical path:

$$\tau(z_{\mathrm{t}}, \lambda) = \sigma(\lambda) N(z_{\mathrm{t}})$$

we get:

$$I(z_{\mathrm{t}}, \lambda) = I_0(\lambda) \exp\left[-\tau(z_{\mathrm{t}}, \lambda)\right]$$

Since there are two different density profiles associated with sunset and sunrise, we can compare the associated differences:

$$I_{\mathrm{sunset}}(z_{\mathrm{t}}, \lambda) - I_{\mathrm{sunrise}}(z_{\mathrm{t}}, \lambda) = I_0(\lambda)\left[e^{-\sigma(\lambda) N_{\mathrm{sunset}}(z_{\mathrm{t}})} - e^{-\sigma(\lambda) N_{\mathrm{sunrise}}(z_{\mathrm{t}})}\right] \quad (1)$$

or:

$$I_{\mathrm{sunset}} - I_{\mathrm{sunrise}} = -2 I_0(\lambda) \exp\left(-\sigma(\lambda) \frac{(N_{\mathrm{sunset}} + N_{\mathrm{sunrise}})}{2}\right) \sinh\left(\sigma(\lambda) \frac{(N_{\mathrm{sunset}} - N_{\mathrm{sunrise}})}{2}\right)$$

We can define an average tangent density $\tilde{N} = (N_{\mathrm{sunset}} + N_{\mathrm{sunrise}})/2$ and an associated 'average' irradiance $\tilde{I} = I_0 \exp\left(-\sigma \tilde{N}\right)$. Furthermore, using the definition of optical thickness, we get:

$$I_{\mathrm{sunset}}(z_{\mathrm{t}}, \lambda) - I_{\mathrm{sunrise}}(z_{\mathrm{t}}, \lambda) = -2\tilde{I}(z_{\mathrm{t}}, \lambda) \sinh\left(\frac{\tau_{\mathrm{sunset}}(z_{\mathrm{t}}, \lambda) - \tau_{\mathrm{sunrise}}(z_{\mathrm{t}}, \lambda)}{2}\right) \quad (2)$$

The interpretation of this equation is straightforward. At low altitudes, the signal difference goes to zero because $\tilde{I}$ goes to zero. At high altitudes, the signal difference goes also to zero since the optical thickness goes to zero. This is the behaviour that is being observed in the LYRA data.

3 Important remark

The previous argumentation of course assumes a general difference in species density between the sunset and sunrise event. As was said during the meeting, this can be caused by the fact that the two events are sampled at two different geolocations. On the other hand, if one signal is systematically lower than the other, this would point to photochemistry that is causing the difference. It would be interesting to investigate this possibility.